What you should learn from Recitation 7: b. More exercise about reduction of order

Fei Qi

Rutgers University

fq15@math.rutgers.edu

March 15, 2014

Disclaimer

- The slides are intended to serve as records for a recitation for math 244 course. It should never serve as any replacement for formal lectures or as any reviewing material. The author is not responsible for consequences brought by inappropriate use.
- There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.

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- In other words, $y_1(t), y_2(t)$ are linearly independent to each other and forms a fundamental set of solutions. The general solution of this ODE would then be

$$y(t) = C_1y_1(t) + C_2y_2(t).$$

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- Then you get $v'(t)$ and by integration you get $v(t)$ and thus $y_2(t) = v_(t)y_1(t)$ and thus the general solution $y(t) = C_1y_1(t) + C_2y_2(t)$.

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Verify that $y_1(t) = e^{-t}$ is a particular solution of the ODE

$$(t+1)y'' + ty' - y = 0$$

and find the general solution

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- Now let $y_2(t) = v(t)y_1(t)$. We formulate the differential equation concerning $v(t)$.

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Divide by e^{-t}:

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Divide by $v'(t)$ and moving things around:

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Divide by $v'(t)$ and moving things around:

$$\frac{v''(t)}{v'(t)} = 2 - \frac{t}{t+1}$$

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Since you are looking for just ONE $y_2(t)$

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\ln v' &= \int (2 - \frac{t}{t+1})dt = \int (2 - \frac{t+1-1}{t+1})dt \\
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Since you are looking for just ONE $y_2(t)$ and hence just ONE $v(t)$,

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$$v'(t) = e^t(1+t)$$

Quiz Problem 1

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$$v(t) = \int e^t(1+t)dt = \int (1+t)de^t = (1+t)e^t - \int e^t d(t+1)$$

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Dr. Z's homework assignment 11 Problem 3a

Verify that $y_1(t) = t^2$ is a particular solution of the ODE

$$3t^2y'' - 12ty' + 18y = 0, t > 0$$

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Dr. Z's homework assignment 11 Problem 3a

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$$y'' - \frac{4}{t}y' + \frac{6}{t^2}y = 0$$

Dr. Z's homework assignment 11 Problem 3a

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$$y(t) = C_1 y_1(t) + C_2 y_2(t) = C_1 t^2 + C_2 t^3.$$

Dr. Z's homework assignment 11 Problem 3b

Verify that $y_1(t) = t$ is a particular solution of the ODE

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$$y'' + \frac{2}{t}y' - \frac{2}{t^2}y = 0$$

Dr. Z's homework assignment 11 Problem 3b

- Formulate the ODE of $v(t)$:

Dr. Z's homework assignment 11 Problem 3b

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Dr. Z's homework assignment 11 Problem 3b

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Dr. Z's homework assignment 11 Problem 3b

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- Solve the ODE and integrate to get $v(t)$

$$\frac{v''}{v'} = -\frac{4}{t}$$

Dr. Z's homework assignment 11 Problem 3b

- Formulate the ODE of $v(t)$:

$$\begin{aligned} & y_1 v'' + (2y_1' + py_1)v' = 0, y_1 = t, p(t) = \frac{2}{t} \\ \Rightarrow\quad & tv'' + (2 + \frac{2}{t} \cdot t)v' = 0 \\ \Rightarrow\quad & tv'' + 4v' = 0 \end{aligned}$$

- Solve the ODE and integrate to get $v(t)$

$$\frac{v''}{v'} = -\frac{4}{t} \Rightarrow \ln v' = -\int \frac{4}{t} dt$$

Dr. Z's homework assignment 11 Problem 3b

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Dr. Z's homework assignment 11 Problem 3b

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- Solve the ODE and integrate to get $v(t)$

$$\frac{v''}{v'} = -\frac{4}{t} \Rightarrow \ln v' = -\int \frac{4}{t} dt = -4 \ln t$$
$$\Rightarrow \quad v' = t^{-4}$$

Dr. Z's homework assignment 11 Problem 3b

- Formulate the ODE of $v(t)$:

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\begin{aligned}
& y_1 v'' + (2y_1' + py_1)v' = 0, y_1 = t, p(t) = \frac{2}{t} \\
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\Rightarrow \quad & tv'' + 4v' = 0
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$$

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Dr. Z's homework assignment 11 Problem 3b

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\Rightarrow \quad & v = -\frac{1}{3} t^{-3}
\end{aligned}
$$

Dr. Z's homework assignment 11 Problem 3b

- Get the general solution:

Dr. Z's homework assignment 11 Problem 3b

- Get the general solution:

$$y_2(t) = v(t)y_1(t)$$

Dr. Z's homework assignment 11 Problem 3b

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Dr. Z's homework assignment 11 Problem 3b

- Get the general solution:

$$y_2(t) = v(t)y_1(t) = -\frac{1}{3}t^{-3} \cdot t = -\frac{1}{3t^2}$$

Dr. Z's homework assignment 11 Problem 3b

- Get the general solution:

$$y_2(t) = v(t)y_1(t) = -\frac{1}{3}t^{-3} \cdot t = -\frac{1}{3t^2}$$
$$\Rightarrow \quad y(t) = C_1 y_1(t) + C_2 y_2(t)$$

Dr. Z's homework assignment 11 Problem 3b

- Get the general solution:

$$
\begin{aligned}
& y_2(t) = v(t)y_1(t) = -\frac{1}{3}t^{-3} \cdot t = -\frac{1}{3t^2} \\
\Rightarrow \quad & y(t) = C_1 y_1(t) + C_2 y_2(t) = C_1 t + C_2 t^{-2}
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Dr. Z's homework assignment 11 Problem 3b

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Remark: the scalar $-1/3$ is now incorporated into the constant C_2.

Dr. Z's homework assignment 11 Problem 3b

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$$

Remark: the scalar $-1/3$ is now incorporated into the constant C_2. This also means you don't have to care about it when you get $v(t)$.

Dr. Z's homework assignment 11 Problem 3c

Verify that $y_1(x) = \sin x^2$ is a particular solution of the ODE

$$xy'' - y' + 4x^3 y = 0$$

and find the general solution.

Dr. Z's homework assignment 11 Problem 3c

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Dr. Z's homework assignment 11 Problem 3c

Verify that $y_1(x) = \sin x^2$ is a particular solution of the ODE

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- $y_1 = \sin x^2, y_1' = \cos x^2 \cdot 2x,$

Dr. Z's homework assignment 11 Problem 3c

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$$xy'' - y' + 4x^3y = 0$$

and find the general solution.

- $y_1 = \sin x^2, y_1' = \cos x^2 \cdot 2x, y_1'' = 2\cos x^2 + 2x \cdot (-\sin x^2 \cdot 2x)$

Dr. Z's homework assignment 11 Problem 3c

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- $y_1 = \sin x^2, y_1' = \cos x^2 \cdot 2x, y_1'' = 2\cos x^2 + 2x \cdot (-\sin x^2 \cdot 2x) = 2\cos x^2 - 4x^2 \sin x^2$.

Dr. Z's homework assignment 11 Problem 3c

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Dr. Z's homework assignment 11 Problem 3c

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$$x(2\cos x^2 - 4x^2 \sin x^2) - 2x\cos x^2 + 4x^3 \sin x^2$$

Verify that $y_1(x) = \sin x^2$ is a particular solution of the ODE

$$xy'' - y' + 4x^3 y = 0$$

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- $y_1 = \sin x^2, y_1' = \cos x^2 \cdot 2x, y_1'' = 2\cos x^2 + 2x \cdot (-\sin x^2 \cdot 2x) = 2\cos x^2 - 4x^2 \sin x^2$. Putting everything into the ODE:

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Dr. Z's homework assignment 11 Problem 3c

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So $y_1 = \sin x^2$ is a particular solution.

Dr. Z's homework assignment 11 Problem 3c

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- Get the standard form

Dr. Z's homework assignment 11 Problem 3c

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and find the general solution.

- $y_1 = \sin x^2, y_1' = \cos x^2 \cdot 2x, y_1'' = 2\cos x^2 + 2x \cdot (-\sin x^2 \cdot 2x) = 2\cos x^2 - 4x^2 \sin x^2$. Putting everything into the ODE:

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  So $y_1 = \sin x^2$ is a particular solution.
- Get the standard form

$$y'' - \frac{1}{x} y' + 4x^2 y = 0$$

Dr. Z's homework assignment 11 Problem 3c

- Formulate the ODE of $v(x)$:

Dr. Z's homework assignment 11 Problem 3c

- Formulate the ODE of $v(x)$:

$$y_1 v'' + (2y_1' + py_1)v' = 0,$$

Dr. Z's homework assignment 11 Problem 3c

- Formulate the ODE of $v(x)$:

$$y_1 v'' + (2y_1' + py_1)v' = 0, y_1 = \sin x^2,$$

Dr. Z's homework assignment 11 Problem 3c

- Formulate the ODE of $v(x)$:

$$y_1 v'' + (2y_1' + py_1)v' = 0, y_1 = \sin x^2, p = -\frac{1}{x}$$

Dr. Z's homework assignment 11 Problem 3c

- Formulate the ODE of $v(x)$:

$$\begin{aligned} & y_1 v'' + (2y_1' + py_1)v' = 0, y_1 = \sin x^2, p = -\frac{1}{x} \\ \Rightarrow \quad & \sin x^2 v'' + (4x \cos x^2 - \frac{1}{x} \cdot \sin x^2)v' = 0 \end{aligned}$$

Dr. Z's homework assignment 11 Problem 3c

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\begin{aligned}
& y_1 v'' + (2y_1' + py_1)v' = 0, y_1 = \sin x^2, p = -\frac{1}{x} \\
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\Rightarrow \quad & \frac{v''}{v'} = \frac{1}{x} - 4x\cot x^2.
\end{aligned}
$$

Dr. Z's homework assignment 11 Problem 3c

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- Integrate both sides to solve v':

Dr. Z's homework assignment 11 Problem 3c

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- Integrate both sides to solve v':

$$
\ln v' \quad = \quad \ln x - \int 4x\cot x^2 dx
$$

Dr. Z's homework assignment 11 Problem 3c

- Formulate the ODE of $v(x)$:

$$y_1 v'' + (2y_1' + py_1)v' = 0, y_1 = \sin x^2, p = -\frac{1}{x}$$

$$\Rightarrow \quad \sin x^2 v'' + (4x\cos x^2 - \frac{1}{x}\cdot\sin x^2)v' = 0$$

$$\Rightarrow \quad \frac{v''}{v'} = \frac{1}{x} - 4x\cot x^2.$$

- Integrate both sides to solve v':

$$\ln v' \quad = \quad \ln x - \int 4x\cot x^2 dx = \ln x - 2\int \cot x^2 dx^2$$

Dr. Z's homework assignment 11 Problem 3c

- Formulate the ODE of $v(x)$:

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\begin{aligned}
& y_1 v'' + (2y_1' + py_1)v' = 0, y_1 = \sin x^2, p = -\frac{1}{x} \\
\Rightarrow \quad & \sin x^2 v'' + (4x \cos x^2 - \frac{1}{x} \cdot \sin x^2)v' = 0 \\
\Rightarrow \quad & \frac{v''}{v'} = \frac{1}{x} - 4x \cot x^2.
\end{aligned}
$$

- Integrate both sides to solve v':

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\begin{aligned}
\ln v' &= \ln x - \int 4x \cot x^2 dx = \ln x - 2 \int \cot x^2 dx^2 \\
&= \ln x - 2 \ln \sin x^2
\end{aligned}
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Dr. Z's homework assignment 11 Problem 3c

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- Integrate both sides to solve v':

$$\begin{aligned} \ln v' &= \ln x - \int 4x\cot x^2 dx = \ln x - 2\int \cot x^2 dx^2 \\ &= \ln x - 2\ln\sin x^2 \end{aligned}$$

(I hope you still remember how to integrate $\cot u$)

Dr. Z's homework assignment 11 Problem 3c

- Formulate the ODE of $v(x)$:

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- Integrate both sides to solve v':

$$\begin{aligned} \ln v' &= \ln x - \int 4x\cot x^2 dx = \ln x - 2\int \cot x^2 dx^2 \\ &= \ln x - 2\ln\sin x^2 \\ &\quad \text{(I hope you still remember how to integrate } \cot u\text{)} \\ &= \ln \frac{x}{(\sin x^2)^2} \end{aligned}$$

Dr. Z's homework assignment 11 Problem 3c

- Integrate to get v:

Dr. Z's homework assignment 11 Problem 3c

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$$\Rightarrow v' = \frac{x}{(\sin x^2)^2}$$

Dr. Z's homework assignment 11 Problem 3c

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$$\Rightarrow v = \int \frac{x}{(\sin x^2)^2} dx$$

Dr. Z's homework assignment 11 Problem 3c

- Integrate to get v:

$$\Rightarrow v' = \frac{x}{(\sin x^2)^2}$$
$$\Rightarrow v = \int \frac{x}{(\sin x^2)^2} dx = \frac{1}{2} \int \frac{1}{(\sin x^2)^2} dx^2$$

Dr. Z's homework assignment 11 Problem 3c

- Integrate to get v:

$$\begin{aligned} \Rightarrow v' &= \frac{x}{(\sin x^2)^2} \\ \Rightarrow v &= \int \frac{x}{(\sin x^2)^2} dx = \frac{1}{2} \int \frac{1}{(\sin x^2)^2} dx^2 \\ &= -\cot x^2 \end{aligned}$$

Dr. Z's homework assignment 11 Problem 3c

- Integrate to get v:

$$\begin{aligned}\Rightarrow v' &= \frac{x}{(\sin x^2)^2} \\ \Rightarrow v &= \int \frac{x}{(\sin x^2)^2} dx = \frac{1}{2} \int \frac{1}{(\sin x^2)^2} dx^2 \\ &= -\cot x^2 \text{ (Knowing this integral is the challenging part)}\end{aligned}$$

Dr. Z's homework assignment 11 Problem 3c

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- Now get $y_2(x)$ and the general solution

Dr. Z's homework assignment 11 Problem 3c

- Integrate to get v:

$$\Rightarrow v' = \frac{x}{(\sin x^2)^2}$$
$$\Rightarrow v = \int \frac{x}{(\sin x^2)^2} dx = \frac{1}{2} \int \frac{1}{(\sin x^2)^2} dx^2$$
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- Now get $y_2(x)$ and the general solution

$$y_2(x) = v(x) y_1(x)$$

Dr. Z's homework assignment 11 Problem 3c

- Integrate to get v:

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- Now get $y_2(x)$ and the general solution

$$y_2(x) = v(x)y_1(x) = -\cot x^2 \sin x^2 = -\cos x^2.$$

Dr. Z's homework assignment 11 Problem 3c

- Integrate to get v:

$$\Rightarrow v' = \frac{x}{(\sin x^2)^2}$$

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$$= -\cot x^2 \text{ (Knowing this integral is the challenging part)}$$

- Now get $y_2(x)$ and the general solution

$$y_2(x) = v(x)y_1(x) = -\cot x^2 \sin x^2 = -\cos x^2.$$

$$y(x) = C_1 y_1(x) + C_2 y_2(x)$$

Dr. Z's homework assignment 11 Problem 3c

- Integrate to get v:

$$\Rightarrow v' = \frac{x}{(\sin x^2)^2}$$

$$\Rightarrow v = \int \frac{x}{(\sin x^2)^2} dx = \frac{1}{2} \int \frac{1}{(\sin x^2)^2} dx^2$$

$$= -\cot x^2 \text{ (Knowing this integral is the challenging part)}$$

- Now get $y_2(x)$ and the general solution

$$y_2(x) = v(x)y_1(x) = -\cot x^2 \sin x^2 = -\cos x^2.$$

$$y(x) = C_1 y_1(x) + C_2 y_2(x) = C_1 \sin x^2 + C_2 \cos x^2.$$

The End